Integrated Flight Modeling: Trajectory Analysis & Hybrid Engine Performance

Team 12 Technical Presentation to the 2018 Spaceport America Cup

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INTRODUCTION

For the 2018 Intercollegiate Rocketry Engineering Competition, the Texas A&M University Sounding Rocketry Team (TAMU-SRT or SRT) will be launching its student-researched and -developed hybrid rocket, Theseus, to an apogee of 10,000 ft above ground level (AGL). In order to achieve this goal, the team must maintain the capability to accurately predict the trajectory of the rocket, given knowledge of the rocket's inertial and aerodynamic properties, as well as realistic assumptions of in-flight conditions. Free, readily-available flight simulation software (OpenRocket, RASAero II, etc.) already exist, however, these programs are often limited in scope. Commercial software often lacks higher-level modeling and simulation capabilities, such as hybrid engine modeling, advanced atmospheric modeling, and landing zone analysis. Additionally, these programs are typically only capable of producing a single nominal flight, without consideration for uncertainty in the rocket's properties and the flight conditions. To solve this problem, TAMU-SRT developed the SRT Flight Simulation (SRT FS). SRT FS is a flexible, easily-extensible, six-degree-of-freedom simulation suite implemented in MATLAB, which utilizes the MonteCarlo method to quantify the statistical uncertainty in the rocket's trajectory. SRT FS is supported by the SRT Hybrid Engine Model (SRT HEM), a robust simulation suite also implemented in MATLAB, which provides engine performance predictions for hybrid rocket engines. SRT FS has been validated with eight flight tests of solid rockets spanning multiple aerodynamic, inertial, and propulsive regimes, resulting in a mean apogee prediction error of only 1.72%.

FLIGHT MODELING

The motion of the rocket is simulated and analyzed in two distinct phases, ascent and descent. During its ascent, the rocket is modeled as a rigid body with six degrees-of-freedom. Standard second-order nonlinear ordinary differential equations (ODEs) are constructed to model the translational and rotational motion during ascent as a function of time; these ODEs are then solved numerically using the Runge-Kutta-Fehlberg Method (RKF45), with a constant step size of 0.01 sec. After reaching apogee, the three rotational degrees-of-freedom are eliminated and the rocket is modeled as a point-mass with only three translational degrees-of-freedom. This is completed with the assumption of a fixed attitude under parachute descent. Similarly to the ascent phase, the ODEs are constructed to model the translational motion during descent and then solved numerically using RKF45.

An aerodynamic database is generated using RASAero II's analysis tool assuming an average skin roughness across the rocket's body, which is approximated from empirical profilometer measurements. Drag coefficient, lift coefficient, and the location of the airframe's center-of-pressure are found as a function of Mach number and total angle-of-attack. Distinct databases are extracted for the powered and unpowered phases of the rocket's flight and are imported into the SRT FS. [2]

An atmospheric model is generated using the U.S. Standard Atmosphere assumptions for isothermal and gradient regions, up to 280,000 ft (84.3 km) above sea level (ASL). Inputs to this model include launch site temperature, pressure, humidity, and elevation. The atmospheric model then outputs temperature, pressure, and density as a function of altitude. Using the dry lapse rate of each atmospheric region, the partial pressure of water vapor is found as a function of altitude, and a density correction for humidity is applied. Wind turbulence is modeled as an altitude-dependent, spatially-frozen three-dimensional vector field, derived from the Dryden power spectral density (PSD) for turbulent intensities.[1] A signal is reconstructed from the Dryden PSD and spatially generated for multiple altitudes to form the continuous gust field model.

ENGINE MODELING

The Theseus launch vehicle is powered by the Nova hybrid engine, developed by SRT. The engine performance and operation are modeled using the SRT Hybrid Engine Model (SRT HEM), a robust simulation suite implemented in MATLAB, which utilizes a forward finite-difference time-stepping scheme to provide hybrid engine performance predictions. SRT HEM has been validated with multiple static engine tests as part of the SRT-4 and SRT-5 (2016-2018) static engine testing campaigns, with peak thrust and total impulse prediction efficiencies of nearly 95%.

An initial fluid state of the nitrous oxide in the oxidizer tank is constructed using a two-phase vapor-liquid equilibrium (VLE) model and initial operating conditions, which include loaded oxidizer mass and temperature or pressure of the oxidizer mixture. An initial engine state is provided by including an engine file, which lists engine parameters, such as the grain and port diameters, fuel density, injection orifice area, nozzle areas, and empirical model coefficients. Given the initial fluid state and initial engine state, SRT HEM then uses a constant step size of 0.01 sec to numerically march the forward finite-difference time-stepping scheme and simulate the pressure-driven flow of oxidizer from the tank into the combustion chamber, the oxidizer injection into the combustion chamber, the combustion of the fuel and oxidizer, and the quasi-one-dimensional flow through the nozzle.

At each time step, a discrete amount of oxidizer is passed from the tank through the injector and into the combustion chamber. The pressure differential across the injector is calculated from an empirical discharge model based on the injection geometry. Then, the discrete injected nitrous oxide gas is coupled with a semi-empirical fuel regression and combustion model based on the mass flux, and combustion of the fuel and oxidizer is simulated. Finally, the quasi-one-dimensional nozzle flow is simulated using a Newton-Raphson solver to determine the Mach number at the nozzle exit plane. The simulation proceeds until the either the flow is unchoked or the fuel or oxidizer has been completely consumed.

Post-processing of the data yields plots, visualizations, and useful engine performance characteristics, including fuel regression rate, flame temperature, oxidizer tank and combustion chamber pressures and temperatures, fuel, oxidizer, and propellant mass flow rates, thrust, exhaust velocity, motor classification, and several more. Using the mass flow rates produced by this model, the mass properties of the rocket – including weight, center-of-gravity, and moments of inertia – are found as a function of time during the operation of the engine and passed to SRT FS.

MONTE CARLO SIMULATION & ANALYSIS

In order to quantify uncertainty in the rocket's trajectory, several hundred random flights are simulated. The initial conditions of each flight are unique and are assigned based on user-specified random statistical distributions. At every time step, random error is inserted into rocket and atmospheric properties; the random error is derived from uniform, normal (Gaussian), or Wiener distributions, with statistical parameters mean (μ), standard deviation (σ), etc., defined by the user.

For the rocket's ascent, the time history of each state variable is processed across the entire set of random flights, yielding a mean behavior. This mean behavior – *the nominal flight* – along with the standard deviation and maximum and minimum values associated with each variable, enables the user to make quantitative assessments about the uncertainty in the nominal trajectory.

For the rocket's descent, the circular-error-probable (CEP) of the set of landing points is used as the primary ballistic analysis tool. A large circular region centered on the mean landing point includes several smaller circular regions within, each of which captures a different probability of the rocket landing within the given circle (CEP25 ≡ 25% probability, CEP50 ≡ 50% probability, etc.). This metric allows the user to quantify the error in the nominal landing point and understand the landing distribution. This landing zone is then projected onto a Google Earth rendering of the launch site and surrounding areas, which enables the user to visualize the landing with regard to local geographic features. The simulation process can be repeated for a range of wind headings and speeds, as well as launch rail elevations and azimuths in order to optimize the launch location and launch rail elevation and azimuth – which serves to mitigate recovery hazards.

VERIFICATION & FURTHER WORK

Several flight tests were carried out in order to evaluate the predictive accuracy of the SRT FS. A total of eight flights of various vehicle sizes, masses, and solid motors were conducted. The SRT FS predicted the apogee of the set of eight flights with an average absolute error of 1.72%. In the future and in order to further improve the accuracy of the simulation, the kinematic model must be extended to allow for the modeling of multi-stage vehicles and off-axis center-of-gravity shifts due to liquid sloshing in the oxidizer tank.

REFERENCE

[1] Beal, T. R., "Digital Simulation of Atmospheric Turbulence for Dryden and von Kármán Models," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, January - February 1993, pp. 132-138.

[2] Rogers, E. C. and Cooper, D., "Rogers Aeroscience RASAero Aerodynamic Analysis and Flight Simulation Software," Available: http://www.rasaero.com/.